Gifted Students in Mathematics Education

**Introduction**

There are some students in the school system, usually by some outstanding stores on standardized tests such as the SATs, have demonstrated some of considerable aptitude for school.  These gifted students are typically capable of learning more effectively and having a modified curriculum for them can capitalize on that.  The focus of this paper will focus on these children’s attributes, potential and interventions meant to capitalize on their said potential.

**The Potential and Future Trajectory of Gifted Students**

Now what is our motivation for modifying our mathematics (or total) curriculum for the gifted student?  Well, longitudinal studies have shown that these students have amazing potential for success in academia, medicine, and business.   (Lubinski, Benbow, Webb, Bleske-Recheck, 2006) (Benbow, Lubinski, Shea, Eftekhari-Sanjani, 2000) and some interventions can improve the chances of having cited publications and help with the acquisition of higher degrees in the sciences, medicine, law.    (Park, Lubinski, Benbow, 2013)

According to work by Benbow Lubinski, Shea, and Eftekhari-Sanjani, students who are in the top percentile of middle school mathematicians are will achieve a bachelor’s 26% of the time and a doctorate 26% of the time, allowing such a small portion of our students to comprise a large section of our most educated.  (Benbow, 2000)  In particular the males would be likely to enter the inorganic sciences and engineering, whereas the females were found to pursue an extremely broad range of disciplines, but medicine and biological sciences in particular.  Amongst this population it was found that males were extremely career driven, while the females were more likely to seek a balance with career family and friends.   In a future study on this group, it was determined that the greatest determining factor for the students’ career trajectory was ability pattern and preference.  (Lubinski, Benbow, 2006a)  This seems to imply that many gifted students can pursue their preferred field if given adequate opportunity.  Interestingly, it was also shown that even those students who decide to major in a non- scientific field, frequently ended up having a profession in a scientific field.  It was also noted that females who had high math ability, typically also had a very high verbal ability.  Lubinski and Benbow emphasize that in today’s multidimensional work environment, that this is an invaluable asset to have and can send them in a variety of different directions.

Lubinksi and Benbow, with the help of Webb and Bleske-Recheck, also investigated the very top tier of the highly gifted when they performed a longitudinal study on a group of 7th and 8th graders that scored in the top .01% on their SATs.  These students represented in a particularly valuable resource to academia, as their success trajectory mirrored that of graduate students that were already in the top universities in the United States. (Lubinski, Benbow, Webb, Bleske-Recheck, 2006)  Doctoral degrees were earned by most students and of these students, most went to top ten universities.  Of those that got MBA’s 95% were at highly ranked schools.  By a wide margin, the most common professions for females in the group was professorship or post-secondary teaching of some kind, followed by executive and administrative positions.  Males in this group were most likely to pursue administration, followed by mathematics, computer science, engineering and professorship.  Of those in tenure track positions, about a third were in the top 25.  Another prominent measure of success is income.  Approximately 30% of the students were had a six figure income, 5% were making over $250,000 per year and about 2% were making over half a million dollars per year, all in their early 30s.  An important measure of productive creativity is patents, and of this population, 17.8% of males and 4.3% of the females had patents, compared to the less than 1% of the general population.  Taken together, this implies that an astonishing amount of the creative and scientific progress in our society, in addition to running much of it, is due to a rather small percentile of the population.  If our education can be optimized to suit these individuals it would very much behoove our society to do so.

**Attributes of a Mathematically Gifted Student**

Before determining what is best for a gifted student, we would like to better understand them.  To improve the education of someone who has an exceptional ability in mathematics, we might wish to make the process enjoyable, in addition to offering new learning opportunities, to keep the student engaged.  Hassan, Ching, Ibrahim and Hamizah have explored this topic with some depth.  They took thirty gifted students from the Pintar National Gifted Centre.  Among these gifted students, they were subdivided into three groups based on their math efficacy.  Despite being gifted, there can be a broad range in how skilled that person in mathematics, so this helps us narrow in on how math efficacy can affect student preferences.  They then performed to separate studies, one for what causes positive attitudes, and one for what causes negative attitudes.  Universal to all groups was that teacher attitude was very important for positive attitudes.  (Hassan, Ching, Hamizah, 2012a)  The high scorers tended to derive their attitudes much more from their grades, but very little from the textbook used.  The low scoring group felt that learning environment really helped to improve their attitude, but got less out of challenging questions.  The second survey indicated that the greatest risks to a good attitude were teacher’s personality and the materials used during class.  (Hasssan, Ching, Ibrahim, 2012b)  It is clear that the teacher’s disposition is exceptionally important for shaping a student’s attitude about a subject, good or bad.  Hassan, Ching and Ibrahim suggest that trying to make a positive relationship with your students can really make a difference due to this.  These studies indicate that students are more concerned with how material is presented in class than how it is in the textbook, so care should be made to have those positive relationships be built on quality presentations of the material.  These results also indicate that some of the curriculum ought to be differentiated, even among the gifted students, since the better students seem to really thrive on the challenge problems.

Not only should be care be taken to gifted students in general, but Lubinski and Bebow’s longitudinal study demonstrates that it is beneficial to be able to adapt to the individual.  (Lubinski, Benbow, 2006a)  Much of the time, a gifted students’ favorite classes will be their AP classes.  Lubinski, et al imply that which AP classes specifically, and their future trajectory is determined more by these proficiencies and preferences than other factors.  Vialle, Ashton, Carlon and Rankin point out that they may not excel at the same rate in certain areas.  (Vialle, Ashton, Carlon, Rankin, 2011)  A successful modified curriculum should be able to accommodate the fact that they may not excel ‘evenly’ across subject and that modified curriculum can create non-academic issues that have to be dealt with, such as emotional and social issues.

**Curriculum Modifications**

Curriculum Acceleration

Having motivated gifted students is wonderful, though we would wish for them to be directed in a way that can make the most of their attitudes and intellect.  Gifted students in particular seem to benefit from curriculum adjusted to their proficiencies and dispositions.  (Frieman, Sriraman, 2008)  There are three common approaches to this:  accelerated curriculum, enrichment, and ability grouping.  (Kim, 2006)  In addition, while uncommon,  a school could offer a curriculum designed specifically for gifted students designed to foster creativity, abstract thought, and/or advanced mathematical concept understanding.  We will call this method a ‘gifted curriculum’  (Karp, 2010)   It is very helpful if a school offers many of these options or has the opportunity to adapt appropriately to the student and some of these options might even overlap out of necessity.  (Vialle, 2001)

One form of acceleration is vertical timetabling.  If a student is consistently performing at a higher grade level or have done particularly well on a standardized exam, it may be assumed that they either know much of the material given in the next year, that they can infer it, or that they can digest it at a rapid pace.  One would think that this process could alienate those who are now in an entirely new and older social group, though Vialle, Ashton, Carlton and Rankin’s findings indicate that this is normally not the case, and can even improve emotional, and social well-being for a student.  (Vialle, 2001)  The grade skipping method tends to be inexpensive, considering that little extra additional resources are taken up, and the academic and social adjustment of the student should be of primary concern.  (Kim, 2006)  If there is a ‘spiral’ style curriculum this also eliminates repetition for a gifted student that may be unnecessary.  Park, Lubinski and Benbow’s study indicates that if a student is gifted, he is likely to have a much more productive career in academia if they are male, gaining more cited publications. The females in this study that were given accelerated studies were much more likely to pursue medicine or law, very prestigious occupations.  (Park, Lubinski, Benbow, 2013) Park, et al postulate that this could be due to an accumulative effect, implying that by ‘accelerating’ the students early, they continue this acceleration through their whole careers.

Vertical timetabling, another form of acceleration, is when instead of modifying the entire curriculum, the student is only exposed to more advanced material in particular courses.  Vialle’s study of students’ experience of vertical timetabling indicated that a prepared student could find a lot of success and positive experiences in this activity.  (Vialle, 2001)  In fact, after an approximate two month adjustment period, these students would excel to the top of their class.  It is noteworthy that a student that skips a grade may find that he is still very bored or unchallenged in a particular class.  Vertical timetabling can ameliorate this situation by targeting the one class that has become disinteresting, making the two plans a good complement in some situations.

Criticism of acceleration

Acceleration is not without its problems.  Whether done with one class in vertical timetabling or all of them in grade skipping, it is not a sequence of that is designed for a student, but a rough adaption to a student who is learns at a fast rate, meaning the student will have to do a lot of work on adaption.  (Kim, 2006)  The skipping grade method implies that there is a huge amount to catch up on at the beginning of the year, perhaps overburdening the student with work, but then they may find that the class has the same pace, and grow bored and dissatisfied after the initial over-acceleration.  In addition, the success of the student in these situations is highly contingent on his or her attitude.  They often have a lot of adapting to do.  (Vialle, 2011)  Some students may even see differentiation in the classroom as unfair and imposing too much on them compared to the other students.  In this instance, while they may be persuaded to do quality work, it may be a very uphill battle for the teacher to convince the student of the program’s worth.  (Reed, 2011)  In addition there is disagreement about whether education for an extremely gifted child should be all about how quickly the person can progress through the curriculum.  (Barbeau, 2011)  This brings us to an alternative type

Enrichment and Differentiation

As mentioned, rather than simply trying to simply racing through curriculum through something like vertical timetabling, some schools may offer a more ‘horizontal’ approach.  In enrichment and differentiation, there is a modified curriculum that involves more fully exploring a concept and giving and finding more challenging and interesting challenges to explore.  (Kim, 2006)  What this will also mean is that the amount of differentiation offered is going to be determined by how much extra effort the teacher is willing to put in.  The students will also be greatly benefited by working together with like-minded students, which leads ability grouping to be an ideal complement to this method.  The biggest danger in this sort of curriculum is that instead of offering additional material that really gets the gears turning in a creative mind that they are just given extra problems that bore rather than stimulate the students.  Instead Freiman and Sriraman encouage bringing in more challenging and thought provoking challenged to engage the student and claim that this has benefits not only for gifted students, but various non-gifted too.  (Frieman, Sriraman, 2008)  They propose using this as an opportunity to foster creativity in mathematics by encouraging a range of solution approaches, requiring justification, having an open ended-ness, and indulging in modeling problems.  This is meant to provide a stimulating environment of invention, guessing, and modifying techniques, instead of just rote problem repetition and formula memorization.

Ability Grouping

(Brulles, Peters, Saunders, 2012)

Ability grouping involves taking students that have similar proficiency in a subject and having them work together.  This permits gifted students to work on activities that are more challenging, advanced, or creative in nature.  This process of differentiation does not have to be exclusionary.  Reed has demonstrated that if a non-gifted student is willing, they can benefit from deciding to join such a group and take part in the more challenging work.  (Reed, 2004) In fact, in Brulles, Peters, and Saunders 2012 study, they demonstrated that non-gifted students could make the same sort of progress in a gifted cluster, while still giving the opportunity for gifted students to advance further. (Brulles, Peters, Saunders, 2012)  The disadvantage of this compared to the prior two methods spoken of, is it typically restricts the rate at which a student can progress to the grade they are currently in (Kim, 2006)  This can be mitigated if a groups stays together over an entire year, allowing unique material that they covered to build on itself.

Gifted Curriculum

Prior methods of modification we have discussed involve taking ordinary curriculum and adjusting it in some way for the student, rather than having it tailored from the start for the gifted student.  A gifted curriculum could be thought of as taking the ideas of differentiation and utterly fleshing them out into an entire curriculum.  Barbeau advises emphasizes that faster is not always better.  The studies she draws from indicate that students would be better served by gaining insight into the higher level thinking, abstraction, and generality of mathematics.  (Barbeau, 2011)  She advises using a learning tool such as geometer's sketchpad for exploration or investigating a variety of abstract subjects that may otherwise stump them when they encounter them for what would be the first time in college, eg. cardinality.  While such programs can be prohibitively expensive, they frequently have significant pay off in terms of student development.  One such curriculum is the Mentoring Mathematical Minds program for 3rd 4th and 5th graders that are mathematically gifted according to the Iowa Test of Basic Skills (ITBS).  This program provides a wealth of rich material on algebra, data analysis, geometry or measurement, numbers, and operations for the advanced student.  This program has shown to be a significant success with both with further increasing scores on the ITBS and with more open assessment procedures.  (Gavin, Casa, Adelson, Caroll, Sheffield, 2009)

One of the most significant examples of this are the Russian specialized mathematics schools.  In this sort of schooling problem based learning is thoroughly emphasized and a proof based course may this so seriously as to give the students nothing but the axioms of a course and have the attempt to derive the rest.  In Karp’s paper, it is emphasized how much material the teachers are essentially forced to go through due to the exceptional wit of the students and, sometimes over three times the predicted pace, and how creative and dynamic they must be to adapt to their class.  (Karp, 2010)  The existence of this school and the students’ exemplary progress indicates a need for this sort of schooling in other places.  If students that are suited for this sort of schooling exist in Russia, then presumably in most other places with adequate education opportunities.  It is worth noting though, that these Russian schools would often spend a disproportionate amount of time on mathematics compared to other subjects, which might be an undesirable sacrifice to many.  Currently, these sort of programs are rare and far between until a student gets to high school, when AP classes are a valid option.

**Conclusion**

It is apparent that gifted education is often tremendously helpful.  It can help transform a student’s experience from boredom and dissatisfaction, fascination and fulfillment while simultaneously helping to provide a future as an academic, doctor, administrator or some other job of his or her choosing.  A common theme in this branch of education is that one size does not fit all.  Students will sometimes need extra attention to maintain their social, emotional health or academic needs during an adjustment period, but a student will normally consider this process a positive experience.  .  (Vialle, et al, 2001)  It is apparent that these programs are essential and should continue to evolve to adapt to students’ needs and give them the best chance to use and enhance their talents.

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